Enrollment No: ______________________ **Exam Seat No:** ____________________

C. U. SHAH UNIVERSITY

Winter Examination-2022

Subject Name: Problem Solving-II

Subject Code: 5SC03PRS1 **Branch: M.Sc. (Mathematics)**

Semester: 3 **Date: 25/11/2022** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions.** **[07]**

a. Classify the region in which the equation $x^2 r - 2s + t = 0$ is hyperbolic. **(02)**

b. Is a group of order 10 simple? Verify. **(02)**

c. Solve: $p^2 - q^2 = x - y$. **(02)**

d. How many different commutative binary operations can be defined on the set $\{a, 1\}$? **(01)**

Q-2 **Attempt all questions** **[14]**

a. Show that a group of order 20449 is abelian. **(06)**

b. With proper justification, prove or disprove:If G is a group of order pq then G has at least one subgroup having order p ,where p, q are prime numbers and $p > q$. Also state the result you use. **(05)**

c. Does there exist a group G with $o\left(\frac{G}{Z(G)}\right) = 79$? Justify. **(03)**

OR

Q-2 **Attempt all questions** **[14]**

a. Find the total number of irreducible monic quadratic polynomials in $Z_P[X]$, where p is prime. **(06)**

b. Let G be a finite abelian group of order n.When the map $\phi: x \to x^m$ be an automorphism? Justify. **(05)**

c. Show that $\{1, -1, i, -i\}$ is an abelian group of order 4 under multiplication. **(03)**

Q-3 **Attempt all questions.** **[14]**

a. Find complete integral of $(p^2 + q^2) = qz$ using charpit's method. **(06)**

b. Solve: $(D^2 - 5DD' + 4D'^2)z = \sin(4x + y)$ **(05)**

c. Show that the polynomial $x^{p^n} - x \in Z_p[x]$ can't have a root with multiplicity greater than 1. **(03)**

OR

Q-3	**Attempt all questions**	**[14]**
a.	Solve $\frac{dy}{dx} = x^2 + y^2$given $y(1) = 1.2$ Find $y(1.05)$ using fourth order Runge Kutta's method (take $h = 0.05$).	**(06)**
b.	Evaluate $f(8)$ using Newton's Divided difference formula from the following table:	**(05)**

x	4	5	7	10	11	12
f(x)	48	100	294	900	1210	2028

c.	Find order of all elements in $U(15)$.	**(03)**

SECTION – II

Q-4	**Attempt the Following questions.**	**[07]**
a.	Let $\alpha = (1\ 3\ 5\ 7\ 9\ 8\ 6)(2\ 4\ 10)$.Find the smallest integer n for which $\alpha^n = \alpha^{-5}$.	**(02)**
b.	Construct a field of order 25.	**(02)**
c.	Find isomorphic group to $U(105)$.	**(02)**
d.	Find $\Delta^5 e^7$.	**(01)**

Q-5	**Attempt all questions**	**[14]**
a.	Solve:$(D - 2D')(D - 3D' + 2)z = e^{2x+y}(1 + xy)$.	**(06)**
b.	Using Euler's Modified method find $y(0.6)$ given $y' = 1 - 2xy$, $y(0) = 0$.Take $h = 0.2$.	**(05)**
c.	Let G be a non-abelian group of order p^3 where p is prime then find $o\big(G/Z(G)\big)$.	**(03)**

OR

Q-5 **Attempt all questions** **[14]**

a. Solve the system of equations **(05)**

$$3x + y - z = 3$$
$$2x - 8y + z = -5$$
$$x - 2y + 9z = 8$$

Using Gauss Elimination method.

b. Find complete integral of $z(xp - yq) = y^2 - x^2$. **(05)**

c. Use Lagrange's Inverse Interpolation Formula to find x when $f(x) = 14$ given $f(0) = 16.35$, $f(5) = 14.88$, $f(10) = 13.59$ and $f(15) = 12.46$ **(04)**

Q-6	**Attempt all questions**	**[14]**
a.	Solve the Heat Equation $\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = \frac{1}{k}\frac{\partial\varphi}{\partial t}$ by the method of separation of variables and show that the solution is of the form $\varphi(x, y, t) = e^{\pm i(nx+my)-(n^2+m^2)kt}$where n and m are some constants.	**(06)**
b.	Find a real root of the equation $x^3 + x^2 - 1 = 0$ using Iteration method.	**(04)**
c.	For which values of n, the polynomial $p(x) = x^3 - nx + 2$ is reducible over **Q**?	**(04)**

OR

Q-6 **Attempt all questions** **[14]**

a. For which values of a the following system of equations have no solution? Exactly one solution? **(06)**

$$x + 2y - 3z = 4\,, 3x - y + 5z = 2\,, 4x + y + (a^2 - 14)z = a + 2$$

b. Find the missing value in the following data: **(05)**

X	1	2	3	4	5
Y	2	5	7	---	32

c. Find $\Delta(e^{ax}\ log\ (logbx))$. **(03)**